Can Quantum de Sitter Space Have Finite Entropy?

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Abstract

If one tries to view de Sitter as a true (as opposed to a meta-stable) vacuum, there is a tension between the finiteness of its entropy and the infinite-dimensionality of its Hilbert space. We invetsigate the viability of one proposal to reconcile this tension using q-deformation. After defining a differential geometry on the quantum de Sitter space, we try to constrain the value of the deformation parameter by imposing the condition that in the undeformed limit, we want the real form of the (inherently complex) quantum group to reduce to the usual SO(4,1) of de Sitter. We find that this forces q to be a real number. Since it is known that quantum groups have finite-dimensional representations only for q = root of unity, this suggests that standard q-deformations cannot give rise to finite dimensional Hilbert spaces, ruling out finite entropy for q-deformed de Sitter.

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Contents

1 Introduction

Observations indicate that our Universe is currently in a regime of accelerated expansion, and that we might be living in an asymptotically de Sitter spacetime [1, 2]. From the perspective of a co-moving observer, de Sitter spacetime has a cosmological horizon and an associated finite entropy [1, 5]. Finiteness of entropy suggests finite dimensionality of the Hilbert space, but since the isometry group of de Sitter is non-compact and therefore has no finite dimensional unitary representations, we immediately have a problem in our hands [4, 3].

One idea that has been proposed as a way out of this quandary is to look for q-deformations [6, 7, 8, 9] of the isometry group which might admit finite dimensional unitary representations. When the deformation parameter(s) are taken to $\rightarrow 1$, we recover the classical group. It is known that the standard q-deformations of (complexified and therefore non-compact) classical groups have finite dimensional unitary representations when q is a root of unity [10, 11, 12, 13, 14, 15, 17].

In this paper, we first look at how the geometric structure of the underlying de Sitter space is modified when its symmetry group is deformed. It turns out that a deformed symmetry group necessitates a deformed differential calculus on the underlying space in order for the differential structure to be covariant with respect to the

new q-symmetry. In section 2, we will make use of the work by Zumino et al. [18, 19] to explicitly write down the differential calculus on the quantum Euclidean space underlying the deformed $SO(5;\mathbb{C})$. Quantum groups are defined through their actions[1] on complex vector spaces, and so we need to start with $SO(5;\mathbb{C})$ before restricting to an appropriate real form to obtain $SO(4,1)$.

To obtain this real form, we need to choose a "$*$-structure" (conjugation) on the algebra [20, 22]. We do this in section 3 and get $SO(4,1)_q$. The definition of conjugation that is necessary for imposing the reality condition on the group elements, induces a conjugation on the underlying quantum space as well. Using this we can choose our co-ordinates to be real, and by imposing an $SO(4,1)_q$-covariant constraint on the quantum space we get a definition of quantum deSitter space. This is analogous to the imposition of $-(X^0)^2 + (X^i)^2 = 1$ on a five-dimensional Minkowski space (thought of as a normed real vector space) to get the classical de Sitter space.

The interesting thing is that the allowed real form of $SO(5;\mathbb{C})_q$ which gives rise to $SO(4,1)$ in the $q \to 1$ limit is constrained by the condition that q be real. But the representation theory of standard quantum groups allows finite-dimensional representations only when the deformation parameter is a root of unity [17, 16]. This suggests that to get finite dimensional Hilbert spaces that could possibly be useful for de Sitter physics, we might need to look for non-standard deformations. Or it might be an indication that quantum mechanics in de Sitter space is too pathological to make sense even after q-deformation.

Finiteness of de Sitter Hilbert space has also been discussed in [27, 28], and q-deformation in the context of AdS/CFT has been considered in [29, 30].

2 Differential Calculus on the Quantum Euclidean Space

Following [18], we will consider deformations of the differential structure of the underlying space (with co-ordinates x^k, $k = 1, 2, ...N$) by introducing matrices B, C and F (built of numerical coefficients) such that

$$B^{kl}_{mn} x^m x^n = 0, \tag{2.1}$$

$$\partial_l x^k = \delta^k_l + C^{km}_{ln} x^n \partial_m, \tag{2.2}$$

[1] To be precise we should say co-actions, but we are using the word loosely.

$$\partial_n \partial_m F^{mn}_{kl} = 0. \tag{2.3}$$

In the limit when there is no deformation, these matrices should tend to the limits

$$B^{kl}_{mn} \rightarrow (\delta^k_m \delta^l_n - \delta^l_m \delta^k_n), \tag{2.4}$$

$$C^{lk}_{nm} \rightarrow \delta^k_n \delta^m_l, \tag{2.5}$$

$$F^{mn}_{kl} \rightarrow (\delta^n_k \delta^m_l - \delta^n_l \delta^m_k), \tag{2.6}$$

so that we have the usual algebra of coordinates and their derivatives. We could also deform the commutation relations for 1-forms and exterior differentials, but these follow straightforwardly from the matrices B, C and F upon imposing natural properties like Leibniz rule etc. So we will not concern ourselves with them here.

To construct a calculus on the space that is covariant under the co-action of a quantum group, we will use the R-matrix of the appropriate quantum group to define our matrices B, C and F. To do this, we first look at the matrix[2] $\hat{R}$, which is related to the R-matrix through $\hat{R}^{ij}_{kl} \equiv R^{ji}_{kl}$. This $\hat{R}$ satisfies the quantum Yang-Baxter equation by virtue of the fact that R does:

$$\hat{R}_{12}\hat{R}_{23}\hat{R}_{12} = \hat{R}_{23}\hat{R}_{12}\hat{R}_{23}. \tag{2.7}$$

It has a characteristic equation of the form:

$$(\hat{R} - \mu_1 I)(\hat{R} - \mu_2 I)...(\hat{R} - \mu_m I) = 0. \tag{2.8}$$

It turns out that we can meet all the consistency requirements that B, C and F should satisfy in order for them to define a consistent deformation, if we set

$$C = -\hat{R}/\mu_\alpha, \tag{2.9}$$

$$B = F = \prod_{\beta(\neq\alpha)} (\hat{R} - \mu_\beta I), \tag{2.10}$$

with some choice of the eigenvalue μ_α. With these definitions, the consistency conditions become automatic because $\hat{R}$ satisfies the Yang-Baxter equation.

The R-matrix for $SO(2n+1)$ [20] looks like

$$\begin{aligned} R &= q\sum_{i\neq i'}^{2n} E_{ii}\otimes E_{ii} + q^{-1}\sum_{i\neq i'}^{2n} E_{ii}\otimes E_{i'i'} + E_{n+1,n+1}\otimes E_{n+1,n+1} + \\ &+ \sum_{i\neq j,j'}^{2n} E_{ii}\otimes E_{jj} + (q-q^{-1})\Big[\sum_{i>j}^{2n} E_{ij}\otimes E_{ji} - \sum_{i>j}^{2n} q^{\rho_i-\rho_j} E_{ij}\otimes E_{i'j'}\Big]. \end{aligned} \tag{2.11}$$

[2] $\hat{R}$ is also often called the R-matrix, but we will not do so to avoid confusion.

For $SO(5)$, $n = 2$, i and j run from 1 to 5, and E_{ij} is the 5×5 matrix with 1 in the (i, j)-position and 0 everywhere else. The symbol $\otimes$ stands for tensoring of two matrices. We define $i' = 6 - i$ and $j' = 6 - j$. The deformation parameter is q. Finally, $(\rho_1, \rho_2, ..., \rho_5) = (3/2, 1/2, 0, -1/2, -3/2)$.

The quantum group is defined in terms of matrices $T = (t_{ij})$ so that $RT_1T_2 = T_2T_1R$ where $T_1 = T \otimes I$ and $T_2 = I \otimes T$. For deforming orthogonal groups we also need to specify a norm that is left invariant under the quantum group elements. This is done through the introduction of the matrix $\hat{C}$ (not to be confused with the C introduced earlier) so that $T^t\hat{C}T = T\hat{C}T^t = \hat{C}$ where (for the specific case of $SO(5)$)

$$\hat{C} = \begin{pmatrix} & & & & q^{-3/2} \\ & & & q^{-1/2} & \\ & & 1 & & \\ & q^{1/2} & & & \\ q^{3/2} & & & & \end{pmatrix}. \tag{2.12}$$

We can use an $SO(5)_q$ invariant constraint of the form $x^t\hat{C}x$ =constant (where $x = \{x^i\}$) to define invariant subspaces of the Euclidean space. With appropriate reality conditions, this can give rise to different signatures in the classical limit.

Using the R-matrix defined above, we can define the $\hat{R}$ matrix for our quantum orthogonal space and it has three distinct eigenvalues: $1/q^4, -1/q$ and q. By explicit computation using (2.9, 2.10) and (2.1, 2.2, 2.3) we find that the choice of μ_α that gives rise to a non-degenerate deformation is $\mu_\alpha = -1/q$. We write down this algebra explicitly in an appendix.

This and the other computations done in this article were implemented using the Mathematica package NCALGEBRA (version 3.7) [26].

3 Choice of Real Form

So far we have worked with complexified groups and their deformations. But since we are interested in $SO(4, 1)$ which is a specific real form of $SO(5; \mathbb{C})$, we need to impose a reality condition on the q-group elements. For that, we need a definition of conjugation ($*$-structure). The $*$-structure on the quantum group will induce a conjugation on the underlying quantum space, and we want our co-ordinates to be real under this conjugation. In a basis where the co-ordinates and the quantum group

elements are real, we can write down the metric $\hat{C}$. If the signature of that metric in the $q \to 1$ limit is $\{- + + + +\}$, we have the real form that we are looking for. This is the program we will resort to, for writing down $SO(4,1)_q$. In this section, we will be working in the single-parameter context.

Using [20, 22], we define a $*$-structure[3] by the relation

$$T^* \equiv D\hat{C}^t T \hat{C}^t D^{-1} \tag{3.1}$$

where

$$D = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} .$$

The idea here is this: from FRT [20], we know of the conjugation $\star$, which is defined by $T^\star \equiv \hat{C}^t T \hat{C}^t$. Since it is known that $\star$ does not lead to the real form that we are looking for, we use the involution D, to create a new conjugation from $\star$. D can be shown to respect all Hopf algebra structures: it is a Hopf algebra automorphism. Before we go further, it should also be mentioned that in order for the conjugation $\star$ to preserve the RTT equations, the R-matrix should satisfy $\bar{R}^{ij}_{kl} = R^{lk}_{ji}$ which works if $q \in \mathbb{R}$.

Using the fact that quantum groups co-act on the quantum space, we can induce a conjugation on the quantum space which turns out to be $x^* = \hat{C}^t D x$. As per our program, our next step would be to find a linear transformation

$$x \to x' = Mx \tag{3.2}$$

$$T \to T' = MTM^{-1} \tag{3.3}$$

such that x', T' are real under their respective conjugations. Under such a transformation, the metric $\hat{C}$ must go to

$$\hat{C} \to \hat{C}' = (M^{-1})^t \hat{C} M^{-1}. \tag{3.4}$$

[3] A conjugation $*$ on a Hopf algebra A is an algebra anti-automorphism, i.e., $*(\eta a.b) = \bar{\eta}(*(b)).(*(a))$ for all $a, b \in A$ and $\eta \in \mathbb{C}$, that also happens to be a co-algebra automorphism, $\Delta(*) = (* \otimes *)(\Delta)$, $\epsilon(*) = \epsilon$ and an involution, $*^2$ =identity.

One can check that an M that can do this is,

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & & 1 \\ & 1 & & 1 & \\ & & i\sqrt{2} & & \\ & i & & -i & \\ i & & & & -i \end{pmatrix}$$

and under it, the matrix $\hat{C}$ goes to

$$\hat{C}' = \begin{pmatrix} \frac{1}{2q^{3/2}} + \frac{q^{3/2}}{2} & 0 & 0 & 0 & \frac{i}{2q^{3/2}} - \frac{iq^{3/2}}{2} \\ 0 & \frac{1}{2q^{1/2}} + \frac{q^{1/2}}{2} & 0 & \frac{i}{2q^{1/2}} - \frac{iq^{1/2}}{2} & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -\frac{i}{2q^{1/2}} + \frac{iq^{1/2}}{2} & 0 & \frac{1}{2q^{1/2}} + \frac{q^{1/2}}{2} & 0 \\ -\frac{i}{2q^{3/2}} + \frac{iq^{3/2}}{2} & 0 & 0 & 0 & \frac{1}{2q^{3/2}} + \frac{q^{3/2}}{2} \end{pmatrix}.$$

It is clear that when $q \to 1$, we get the correct signature with one negative and four positive eigenvalues. So we finally have a complete definition of our quantum de Sitter space.

We note here that it is not obvious that all the real forms of the classical groups exist after q-deformation. Twietmeyer [23] and Aschieri [22] have classified the possible real forms of $SO(2n+1)_q$, and they find that for real values of q there are 2^n real forms (our de Sitter belongs to this category), and for $|q| = 1$ there is only one real form, namely $SO(n, n+1)_q$. Since we expect to get finite dimensional representations of quantum groups only when q is a root of unity[4], for $SO(5)$, these are allowed only for anti-de Sitter space [13, 14, 15]: with isometry group $SO(2, 3)$. DeSitter symmetry group can occur only if we choose q to be real, as we saw explicitly.

As already stressed, it might be possible to skirt this issue by working with more generic (multi-parametric or otherwise) deformations and corresponding $*$-structures. But we will not be pursuing those lines here. Maybe de Sitter should only be looked at as a resonance or a metastable state in some fundamental theory like string theory.

[4]At other values of q, the representation theory of quantum groups is "pretty much isomorphic" to the representation theory of classical groups.

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5 Appendix

In this appendix we explicitly write down the form of the $SO(5)_q$-covariant differential calculus with $\mu_\alpha = -1/q$ as our chosen eigenvalue. The technology is well-known in the literature, we write down the explicit form here so we can make some comments.

We first write down the commutators between the coordinates x^i where i goes from 1 to 5.

$$x^1x^2 = \frac{1}{q}x^2x^1,\ x^2x^5 = \frac{1}{q}x^5x^2 \quad (5.1)$$

$$x^1x^4 = \frac{1}{q}x^4x^1,\ x^4x^5 = \frac{1}{q}x^4x^5 \quad (5.2)$$

$$x^1x^3 = qx^3x^1,\ x^2x^3 = qx^3x^2,\ x^3x^4 = qx^4x^3,\ x^3x^5 = qx^5x^3 \quad (5.3)$$

$$q(x^1x^5 - x^5x^1) + (q-1)(x^2x^4 + qx^4x^2) + q^{1/2}(q-1)x^3x^3 = 0 \quad (5.4)$$

$$q(q-1)(x^1x^5 - x^5x^1) + (1-q+q^3)x^2x^4 - q^2x^4x^2 + q^{5/2}(q-1)x^3x^3 = 0 \quad (5.5)$$

$$q^{3/2}(x^1x^5 - x^5x^1) + q^{1/2}(q^2x^2x^4 - x^4x^2) + (q^3 - q^2 + q - 1)x^3x^3 = 0 \quad (5.6)$$

$$q^{3/2}(q-1)(x^1x^5 - x^5x^1) - q^{3/2}x^2x^4 + q^{1/2}(1-q^2+q^3)x^4x^2 - (q-1)x^3x^3 = 0 \quad (5.7)$$

Some of the relations above are redundant. Also, these relations should be thought of in conjunction with the condition that $x^t\hat{C}x = \frac{3}{\Lambda}$ which (with the appropriate reality condition) is the embedding corresponding to de Sitter space. Here Λ could be interpreted as the cosmological constant. One can rewrite the relations above by making use of this constraint and eliminating x^3x^3. For instance, after some algebra

(5.6) becomes

$$\left(\frac{1}{q^2}x^1x^5 - q^2x^1x^5\right) + \left(\frac{1}{q}x^2x^4 - qx^4x^2\right) = \frac{\Lambda(1-q^4)}{3q^{1/2}(1+q^3)}. \tag{5.8}$$

It should be noted that these deformed commutators constructed a 'la Zumino, are the same as the ones written down by [20] and [24]. Their prescription was to split the R-matrix into projection operators so that

$$\hat{R} \equiv qP_S - q^{-1}P_A + q^{-4}P_1, \tag{5.9}$$

and use that to define the deformations according to $P_A(\mathbf{x}\otimes\mathbf{x}) = 0$. The projectors are onto the eigen-subspaces, so everything is consistent.

The algebra of the partial derivatives (which is controlled by the matrix F) can be obtained from the co-ordinate algebra above if we substitute $x^i \to \partial_{x^{i'}}$, where $i' = 6 - i$.

To complete the definition of the deformed calculus, we need to spell out the algebra that deforms the commutators between the co-ordinates and derivatives. It turns out that they are, for x^i and ∂_{x^j} with $i = j$,

$$\partial_{x^1}x^1 = 1 + q^2x^1\partial_{x^1} + (q^2-1)(x^2\partial_{x^2} + x^3\partial_{x^3} + x^4\partial_{x^4}) + \left(1 - \frac{1}{q^3}\right)(q^2-1)x^5\partial_{x^5} \tag{5.10}$$

$$\partial_{x^2}x^2 = 1 + q^2x^2\partial_{x^2} + (q^2-1)(x^3\partial_{x^3} + x^5\partial_{x^5}) + \left(1 - \frac{1}{q}\right)(q^2-1)x^4\partial_{x^4} \tag{5.11}$$

$$\partial_{x^3}x^3 = 1 + qx^3\partial_{x^3} + (q^2-1)(x^4\partial_{x^4} + x^5\partial_{x^5}) \tag{5.12}$$

$$\partial_{x^4}x^4 = 1 + q^2x^4\partial_{x^4} + (q^2-1)x^5\partial_{x^5} \tag{5.13}$$

$$\partial_{x^5}x^5 = 1 + q^2x^5\partial_{x^5} \tag{5.14}$$

and for $i \neq j$,

$$\partial_{x^1}x^2 = qx^2\partial_{x^1} + \left(\frac{1}{q^2} - 1\right)x^5\partial_{x^4} \quad , \quad \partial_{x^2}x^1 = qx^1\partial_{x^2} + \left(\frac{1}{q^2} - 1\right)x^4\partial_{x^5} \tag{5.15}$$

$$\partial_{x^1}x^3 = qx^3\partial_{x^1} + \frac{1-q^2}{q^{3/2}}x^5\partial_{x^3} \quad , \quad \partial_{x^3}x^1 = qx^1\partial_{x^3} + \frac{1-q^2}{q^{3/2}}x^3\partial_{x^5} \tag{5.16}$$

$$\partial_{x^1}x^4 = qx^4\partial_{x^1} + \frac{1-q^2}{q}x^5\partial_{x^2} \quad , \quad \partial_{x^4}x^1 = qx^1\partial_{x^4} + \frac{1-q^2}{q}x^2\partial_{x^5} \tag{5.17}$$

$$\partial_{x^2} x^3 = q x^3 \partial_{x^2} + \frac{1-q^2}{q^{1/2}} x^4 \partial_{x^3} \quad , \quad \partial_{x^3} x^2 = q x^2 \partial_{x^3} + \frac{1-q^2}{q^{1/2}} x^3 \partial_{x^4} \tag{5.18}$$

$$\partial_{x^2} x^5 = q x^5 \partial_{x^2} \quad , \quad \partial_{x^5} x^2 = q x^2 \partial_{x^5} \tag{5.19}$$

$$\partial_{x^3} x^4 = q x^4 \partial_{x^3},\ \partial_{x^3} x^5 = q x^5 \partial_{x^3} \quad , \quad \partial_{x^4} x^3 = q x^3 \partial_{x^4},\ \partial_{x^5} x^3 = q x^3 \partial_{x^5}, \tag{5.20}$$

$$\partial_{x^4} x^5 = q x^5 \partial_{x^4} \quad , \quad \partial_{x^5} x^4 = q x^4 \partial_{x^5} \tag{5.21}$$

$$\partial_{x^1} x^5 = x^5 \partial_{x^1},\ \partial_{x^5} x^1 = x^1 \partial_{x^5} \quad , \quad \partial_{x^2} x^4 = x^4 \partial_{x^2},\ \partial_{x^4} x^2 = x^2 \partial_{x^4}. \tag{5.22}$$

This completes the definition of the $SO(5)_q$-covariant calculus on the quantum space.

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